

# Probabilistic analysis of payback period for AC–DC transmission and distribution asset expansion projects

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**Abstract:** In this study, a new concept of statistical distribution of payback period is calculated for AC–DC transmission and distribution asset expansion projects. For calculation of the payback period, uncertainty is considered with cost and benefit. The Gaussian distribution is considered for correlated input cost and benefit variates. For more examination point of view, the distribution of the payback period is calculated with different correlation coefficients between cost and benefit. The main advantage of this study is to draw the probability distribution function of the payback period in terms of cost and benefit because most of the industrialists are often interested in cost and benefit analysis of AC–DC transmission and distribution asset expansion projects. An example is considered for evaluation of the proposed model.

## 1 Introduction

The future electric power system is designed for efficiency, reliability, ease of operation, and to meet consumer needs at a minimum cost. The grid of the future must maintain these characteristics, such as supporting the integration of various clean and distributed energy technologies, meeting the higher power-quality demands of modern digital devices, and enabling consumer participation in electricity markets. Increasing the projected penetration levels of variable renewable resources, distributed generation, community energy storage, electric vehicles, and the number of active customers will require substantial changes to how the grid and its various components are designed, controlled, and protected [1].

For mitigation of customer needs, it is essential to replace, expand, and upgrade the electric power system components at continuous bases. Investment needed for upgradation of these components included the cost for telecom sector, cost for post data collections system (i.e. data collection process, forensic analysis, and programme benefit analysis), cost for vegetation management programme (i.e. hazard and danger trees and trimming cycles), cost for ground-based inspection programmes, cost for infrastructure hardening programmes (i.e. substations, telecom central offices, back-up power for central offices and substations, hardened transmission structure, non-wood structures for new transmissions, underground transmissions, underground distributions, and targeted storm hardening), and cost of technologies (i.e. technologies for transmissions, technologies for distributions, section applications, communication technologies, logistic management, work scheduling system, and impact of technologies on system restoration time) [2].

Most of the industrialists are interested in cost and benefit analysis for expansion, operation, and planning of electric power transmission and distribution system. For this cost and benefit analysis, different tools are adopted. Around these tools, the power industry had built resources based on personal data and analytical capability, and investments are huge. Most of these developed tools are based on deterministic assessment capability only [3, 4]. For example,  $N-1$  and  $N-2$  are major deterministic techniques for transmission planning and asset expansion in an economical and reliable manner. In this case, the deterministic analysis does not differentiate between the occurrences of individual contingencies

but only focus on the impact of contingencies. Deterministic results fit well in a decision-making process for a deterministic framework. Most of the times, the regulators and industrialists are interested in probabilistic analysis [3]. A probabilistic analysis can handle more flexible and accurately uncertainties with input parameters and can give statistical distribution of results [5].

Recently, different approaches have been used for analysis of transmission and distribution expansion projects, such as for contingency analysis with security-constrained commitment with generation and transmission [6–8], for analysis and prediction of vulnerability in a smart power transmission system [9], probabilistic transmission expansion planning for wind power penetration and its impact [10–12], a multi-objective expansion planning approach [13, 14], by considering planning uncertainty [15], risk-/investment-driven base planning [16], test systems and mathematical models based expansion planning [17], approximations in power transmission planning [18], and a number of case studies performed for probabilistic transmission planning and expansion based on TRANSCARE [19]. In all of the above-mentioned cases, no one addresses the probabilistic application with cost and benefit analysis. So, that is why in this work the cost-to-benefit analysis is proposed in the probabilistic term. Most of the industrialists/regulators are interested in the cost-to-benefit analysis of engineering projects. During the cost and benefit analysis, many parameters are uncertain, and it appears to be very useful when considering uncertainty within cost and benefit parameters. Although uncertainty can be created with other economic parameters, but here only two parameters, i.e. cost and benefit, are taken for illustration of the payback period. These two parameters are modelled as Gaussian variates. The payback period is the ratio between cost and benefit in dollars per year. The main advantage of this model is to compute the probability distribution function (PDF) of the payback period and the probability of exceeding the specific duration of the payback period. An example is considered for analytical point of view; it is related to the transmission asset expansion project. The term AC–DC means that this model can be implemented to AC or DC transmission and distribution system asset expansion projects effectively, by simply changing the parameters.

The rest of the paper is organised as follows: Section 2 introduces the probabilistic modelling of the payback period, cost, and benefit. Section 3 introduces one example for obtaining the

results of the proposed model. Section 4 describes the results and discussion, and finally, Section 5 concludes this research work.

## 2 Probabilistic modelling of the payback period, cost, and benefit

In this work, the payback period is considered as a ratio of bivariate correlated Gaussian-distributed random variables as explained by Hinkley in 1969 [20]. ‘In a regression analysis of bivariate of two correlated data it is sometimes of interest to estimate the ration of two population parameters’. Two examples are as follows:

- i. The analysis of a simple linear model  $y_i = \alpha + \beta_i u_i + \varepsilon_i (i = 1, \dots, n)$ , where  $\varepsilon_1, \dots, \varepsilon_n$  are independently normally distributed with zero mean and variance  $\sigma^2$ ; then,  $-\alpha/\beta$  is the intercept of the regression line with the  $\mu$ -axis.
- ii. The analysis of the two-line linear model is given in the following equation:

$$y_i = \begin{cases} \alpha_1 + \beta_1 u_i + \varepsilon_i & (i = 1, \dots, n_1), \\ \alpha_2 + \beta_2 u_i + \varepsilon_i & (i = n_1 + 1, \dots, n_1 + n_2), \end{cases} \quad (1)$$

with notation as in (i): the ratio is the abscissa of the intersection of the two regression lines. Similarly, in this work, the payback period is a ratio between total investment cost ( $C$ ) and annualised benefit ( $B$ ) as given in the following equation:

$$Y = \frac{C}{B} \quad (2)$$

where  $C$  and  $B$  are the random variables, which follow a Gaussian distribution. These assumptions almost follow the central limit theory. According to the central limit theory ‘the variates are the sum of a large number of random variables that are independently and identically distributed’. So, cost and benefit are the two random variates that fulfil this primary requirement. Additionally, Gaussian distribution is used to model  $C$  and  $B$  because it is a well-known distribution and is easy to implement.

For obtaining the PDF of correlated  $C$  and  $B$ , Gaussian Copula theory is used to generate the correlated samples. The PDF of the payback period is computed by using as the following equation:

$$f_Y(Y) = \frac{bd}{\sqrt{2\pi}\sigma_C\sigma_B a^3} \left[ \operatorname{erf}\left(\frac{b}{a\sqrt{2(1-p^2)}}\right) - \operatorname{erf}\left(\frac{-b}{a\sqrt{2(1-p^2)}}\right) \right] + \frac{\sqrt{1-p^2}}{\pi\sigma_B\sigma_C a^2} \exp\left(\frac{-c}{2(1-p^2)}\right) \quad (3)$$

where  $a$ ,  $b$ ,  $c$ ,  $d$ , and error function (erf) parameters are computed using the following equations, respectively:

$$a = \sqrt{\frac{Y^2}{\sigma_C^2} - \frac{2\rho Y}{\sigma_C\sigma_B} + \frac{1}{\sigma_B^2}} \quad (4)$$

$$b = \frac{\mu_C Y}{\sigma_C} - \frac{\rho(\mu_C + \mu_B Y)}{\sigma_C\sigma_B} + \frac{\mu_B}{\sigma_B^2} \quad (5)$$

$$c = \frac{\mu_C^2}{\sigma_C^2} - \frac{2\rho\mu_C\mu_B}{\sigma_C\sigma_B} + \frac{\mu_B^2}{\sigma_B^2} \quad (6)$$

$$d = \exp\left(\frac{b^2 - ca^2}{2(1-p^2)a^2}\right) \quad (7)$$

$$\operatorname{erf}(\lambda) = \frac{2}{\sqrt{\pi}} \int_0^\lambda \exp(-t^2) dt \quad (8)$$

where  $\mu_C$  and  $\mu_B$  are the mean values of cost and benefit, respectively, and  $\sigma_C$  and  $\sigma_B$  are the variances of cost and benefit, respectively.  $\rho$  is used to show the correlation coefficient between  $C$  and  $B$ . Four different cases are used to represent the best reflection in PDF of the payback period. In four different cases, the statistical correlation between  $C$  and  $B$  is varied from weak to strongly correlated as follows:  $\rho = 0.1, 0.4, 0.8, 0.98$ . The value of  $\rho$  varies largely due to non-stationary distribution of  $C$  and  $B$ , and may be attributed to the cost of money and inflation. It is useful when  $\rho$  varies in a specific range. For computation of PDF of the payback period, it needs to estimate the total investment cost and annual benefit for any certain project. All of the historical investment costs, price trends, and annual benefit data are taken from the proposed example. After computing the probability of  $Y$ , various statistical parameters of  $Y$  can be computed, such as mean and variance by using (9) and (10), respectively, and also the conditional expected value of  $Y$  can be found by (11):

$$E(Y) = \mu_Y = \int_{-\infty}^{\infty} Y f_Y(Y) dY \quad (9)$$

$$E((Y - \mu_Y)^2) = \sigma_Y^2 = \int_{-\infty}^{\infty} (Y - \mu_Y)^2 f_Y(Y) dY \quad (10)$$

$$E(Y|Y \geq \bar{Y}) = \int_{\bar{Y}}^{\infty} Y f_Y(Y) dY \quad (11)$$

## 3 Example

In this work, a report prepared by Richard Brown (Quanta Technology, USA) on behalf of ‘Public Utility Commission (PUC) of Texas’ with the title ‘Cost-benefit analysis of the deployment of utility infrastructure upgrades and storm hardening programs’ has been adopted as an example. The executed summary of this report is ‘examination of cost, utility benefit, and societal benefit for a variety of storm hardening programs’. Based on the data provided by the utilities and other assumptions, the programmes presented in Table 1 have been used for the cost–benefit analysis. There are many programmes that have been used for analysing the cost–benefit analysis for an electric power system against hurricane and storm damage, such as vegetation management, ground-based petrol, telephone central offices, infrastructure hardening, and smart grid technology. However, in this work, only a few programmes have been considered for analysing the PDF of the payback period.

### 3.1 Infrastructure hardening programme

In this programme, a 100-year floodplain has been adopted for analysing the cost–benefit analysis. In this 100-year floodplain, four major components of the electric power system have been upgraded. These major four components are needed to be construction/or expansion substations, construction of new telephone centre offices, hardened transmission lines, and a structure above the 100-year flood elevation.

**3.1.1 Cost–benefit analysis of substation:** In this section, the cost–benefit analysis of a new substation and its net benefit have been discussed for the 100-year floodplain. As shown in Table 1, the cost of constructing a new substation is \$6,000,000 and its repairing cost is \$2,000,000. Moreover, \$16,000 per year is the benefit of upgrading and the total net benefit of this project is \$156,465 with a 10% discount rate and a 40-year substation life.

**3.1.2 Cost–benefit analysis of telephone central office:** In this section, the cost–benefit analysis of designing and constructing a new/or expanding telephone central office has been computed, which accounts to \$1,500,000 and its repairing cost is \$500,000, as

presented in Table 2. From Table 2, it is shown that \$4,000 per year is the benefit of this project. The net benefit of this complete project is \$39,116 with a 10% discount rate and a 40-year structure building life, the same as the substation case.

**3.1.3 Hardened transmission line and structure:** In this section, the cost–benefit analysis of a new transmission line and/or replacing an existing structure design has been considered. Although many factors are considered for designing and construction of transmission lines, but only wind loading criteria have been considered in this work. The old transmission lines and structures were according to the National Electric Safety Code (NESC) ‘Extreme wind and ice’ criteria from ASCE 7-98, ‘minimum design loading for buildings and other structure’. The old structure was designed for over 18 m (60 ft) above ground and 105 mph wind loading criteria. Now, the given structure is needed to be upgraded for 130mph wind loading to fulfil the requirements of NESC extreme wind loading criteria. A few statistics about the transmission line and structure are mentioned here, such as the average cost of the transmission line is \$459,000 per mile, and \$60,000 per structure. Table 3 presents the values computed for different utilities presented in this report [2]. For the cost–benefit analysis point of view, there are many assumptions that have been

considered in this report [2], but a few of them are listed as follows:

- i. The number of structure is proportional to the mile of transmission line within the coastal region.
- ii. Hurricane probability of occurrence is commutated based on hurricane simulation model.
- iii. The average direct cost of per structure is \$60,000.

Table 4 presents the values computed based on the metropolitan statistical utility company Victoria. In this utility company, a total of 65 substations, 8 telephone central offices, 477-mile transmission lines, and 3559 transmission structures are needed to be expanded and upgraded. All data related to this utility company have been used for the cost–benefit analysis.

## 4 Results and discussion

This example ‘Cost-benefit analysis of the deployment of utility infrastructure upgrades and storm hardening programs’ has five key investment costs with their mean and standard deviation values shown in Table 5. The total annual benefits of this project are presented in Table 6 along with their mean and standard deviation values. *C* and *B* data are modelled as correlated Gaussian variates.

**Table 1** Substation cost–benefit analysis

Name of programme	Cost	Benefit
new substation	\$6,000,000	\$6,000,000
probability of damage in floodplain (100 yr flood)	1.0%	—
probability of damage outside floodplain (500 yr flood)	—	0.20%
repairing cost if flooded	\$2,000,000	\$2,000,000
expected annual value of flood repairing cost	\$20,000	\$4000
PV of repair cost of 40 yr life of substation (@10%)	\$195,581	\$39,116
net benefit		\$156,465

**Table 2** Cost–benefit analysis of telephone central offices

Name of programme	Cost	Benefit
new telephone central offices	\$1,500,000	\$1,500,000
probability of damage in floodplain (100 yr flood)	1.0%	—
probability of damage outside floodplain (500 yr flood)	—	0.20%
repairing cost if flooded	\$500,000	\$500,000
expected annual value of flood repairing cost	\$5000	\$1000
PV of repair cost of 40 yr life of substation (@10%)	\$48,895	\$9779
net benefit		\$39,116

**Table 3** Cost–benefit of hardened transmission lines and structures

Utility	Weighted savings damage reduction (\$)	Weighted savings damage reduction (\$)	Weighted savings damage reduction (\$)	Weighted savings damage reduction (\$)	Weighted savings damage reduction (\$)
Entergy (Beaumont-Port Arthur)	2050	6060	1,064,850	106,850	131
CenterPoint & TNMP (Houston)	863	40,690	1,274,271	127,847	31
AEP (Victoria) 20%	202	620	250,400	25,123	304
AEP (Corpus & Brownsville) 80%	2691	3450	1,001,600	100,490	163

**Table 4** Cost–benefit of Victoria utility hardened transmission lines and structures

Name of programme	Cost	Benefit
cost of transmission line and structure upgrade and replacing (50 mile)	\$23,000,000	\$23,000,000
probability of damage in hurricane (10 yr)	1.0%	—
probability of damage outside hurricane (500 yr)	—	0.20%
repairing cost (10 yr)	\$120,000	\$120,000
expect annual value of repairing line and structure	\$60,000	\$12,000
PV of repair cost of 60 yr life of transmission and structure (@10%)	\$1,064,850	\$106,836
net benefit		\$76,408

**Table 5** Cost for Victoria utility expansion project at PUC of Texas

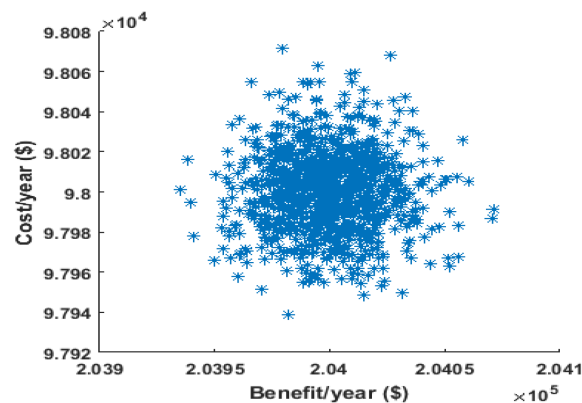
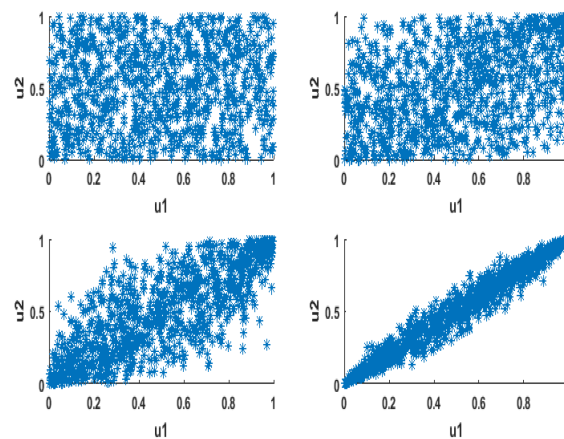
Components	Mean unit cost (\$, per year/each)	Number of units	Standard deviation (\$, each)
substations	6,000,000	65	5000
telephone central offices	1,500,000	8	1600
OH transmission line (mile)	459,000	477	350
transmission structure	61,000	3559	300
installation	500,000	1	60,000

Mean cost  $\mu_C = \$2.04 \times 10^5$  per year;  $\sigma_C = \$23,337$ .

**Table 6** Benefit for Victoria utility expansion project at PUC of Texas

Benefit	Mean unit benefit (\$, per year/each)	Number of units	Standard deviation (\$, each)
substations	6,000,000	65	5000
telephone central offices	1,500,000	8	1600
OH transmission line (mile)	1510	477	600
transmission structure	201	3559	201
miscellaneous	15,000	1	15,000

Mean cost  $\mu_C = \$9.8 \times 10^4$  per year;  $\sigma_C = \$5524$ .

**Fig. 1** Scatter plot for independent generated total cost and benefit/year samples**Fig. 2** Scatter plot of correlated cost and benefit

(a)  $\rho = 0.1$ , (b)  $\rho = 0.4$ , (c)  $\rho = 0.8$ , (d)  $\rho = 0.98$

Fig. 1 shows the almost independently generated samples of  $C$  and  $B$  with mean and standard deviation values shown in Tables 5 and 6. Fig. 2 shows the correlated generated samples with different values of correlation in a unit domain with the help of the Gaussian Copula theory as in MATLAB. PDF and cumulative density function (CDF) of the total investment cost with different values of correlation coefficient are shown in Figs. 3 and 4, respectively. Similarly, PDF and CDF of total investment annual benefit with different values of correlation coefficient are shown in Figs. 5 and 6, respectively. The results of PDF of the payback period  $Y$  are shown in

Fig. 7, which is computed with the help of (3) with different correlation coefficient values. From these results, it is clearly

shown that the PDFs of the payback period are not highly sensitive with variables  $C$  and  $B$ , which are correlated Gaussian distributed variates. However, from the experiment, it is shown that cost and benefit can be modelled in the probabilistic term, which perhaps will not be more sensitive with other types of distributions rather than Gaussian distribution. In fact, it is an ongoing research topic and can be a more effective tool in the future for cost and benefit analysis for distribution and transmission asset expansion projects.

The conditional probability of exceeding a specific duration of the payback period, i.e. when the value of  $Y$  is greater than or equal to the expected value of  $Y$  when  $Y \geq \bar{Y}$ , is also presented in Table 7. In Table 7, results are tabulated when the condition

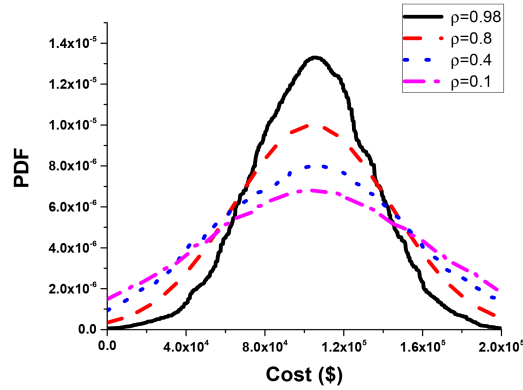


Fig. 3 Probability density function of total cost with different values of  $\rho$

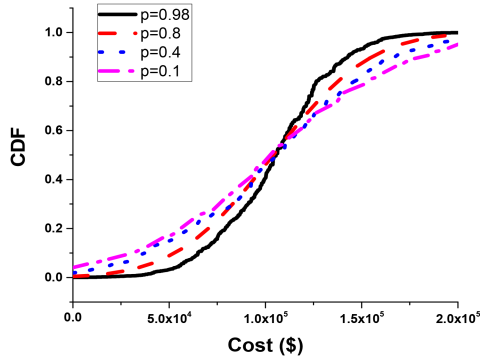


Fig. 4 Cumulative density function of total cost with different values of  $\rho$

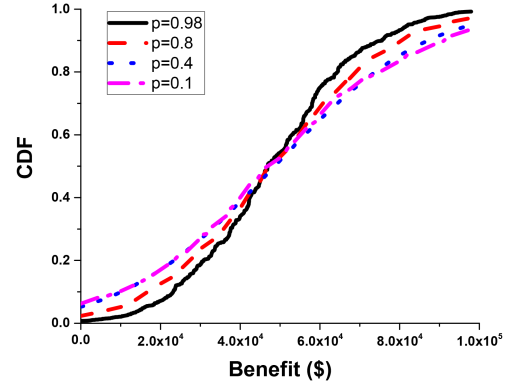


Fig. 6 Cumulative density function of total benefit with different values of  $\rho$

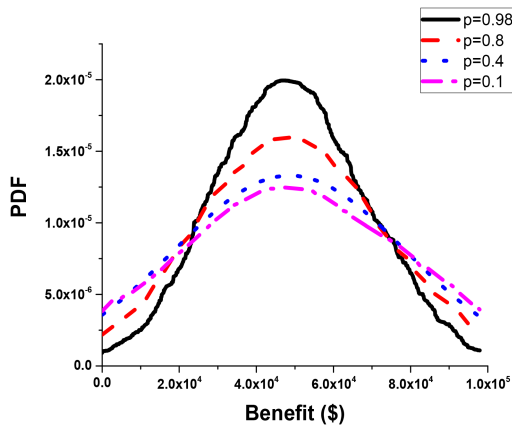


Fig. 5 Probability density function of total benefit with different values of  $\rho$

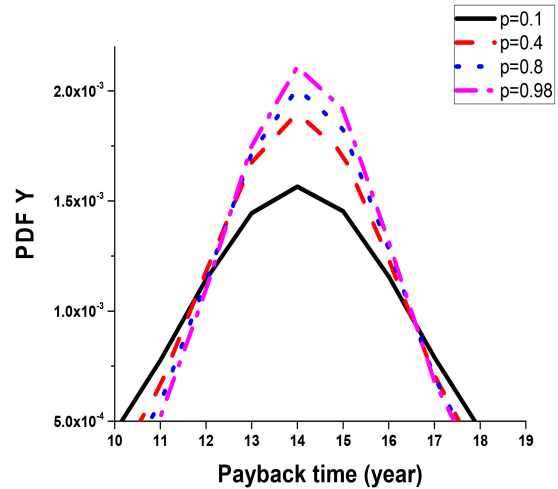


Fig. 7 Probability density function of the payback period ( $Y$ ) of cost and benefit analysis with different values of  $\rho$

$\Pr\{Y \geq 17 | Y \geq 14\}$  exists. These probabilistic measurements are not easily computable with deterministic cost and benefit analysis.

## 5 Conclusion

In this work, a cost and benefit analysis for transmission and distribution asset expansion projects in the probabilistic term has been proposed. For this analysis, two factors, i.e. cost and benefit, are taken for evaluation of the payback period for a given project. Cost and benefit are modelled as correlated Gaussian variates. From the results, it is shown that cost and benefit can be formulated in the probabilistic term for any engineering project. In this way, the payback period becomes a random variable that is very helpful for transmission and distribution asset expansion projects because most of the industrialists are interested in the payback period. Additionally, statistical distribution of the payback period has become quite helpful for decision makers as well. More, various statistical measures like the expected payback period, the standard deviation in years, and the conditional probability to exceed a specific duration of the payback period are also possible in this way.

J. Eng.

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**Table 7** Probabilistic indices for cost and benefit analysis

Quantity measure	Correlation coefficient			
	0.1	0.4	0.8	0.98
mean value of $Y$ (yr)	14.02	14.05	14.12	14.16
standard deviation of $Y$ (yr)	2.55	2.1	1.98	1.88
conditional probability ( $Y \geq 14$ ) (yr)	0.484	0.492	0.516	0.572
$\Pr \{Y \geq 17   Y \geq 14\}$	$8.2 \times 10^{-4}$	$6.7 \times 10^{-4}$	$6.6 \times 10^{-4}$	$6.5 \times 10^{-4}$

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